

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS
AS LEVEL
H630/01
MATHEMATICS B (MEI)
Pure Mathematics and Mechanics
Question Paper
WEDNESDAY 16 MAY 2018: Morning
TIME ALLOWED: 1 hour 30 minutes
plus your additional time allowance
MODIFIED ENLARGED 36pt**

YOU MUST HAVE:

**Printed Answer Booklet sent with the
standard paper. The centre may enlarge the
Printed Answer Booklet.**

Insert for Questions 4 and 11(iii)

YOU MAY USE:

a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS

Use black ink. HB pencil may be used for graphs and diagrams only.

Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number or write them on the paper provided by the centre.

Answer ALL the questions.

IF YOU USE THE PRINTED ANSWER BOOKLET, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

The total number of marks for this paper is 70.

The marks for each question are shown in brackets [].

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots \\ + {}^nC_r a^{n-r} b^r + \dots + b^n$$

$$(n \in \mathbb{N}), \text{ where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \dots \\ + \frac{n(n-1)\dots(n-r+1)}{r!} x^r + \dots$$

$$(|x| < 1, n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where}$$

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$

where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Answer ALL the questions.

1 Write $\frac{8}{3 - \sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers to be found. [2]

2 Find the binomial expansion of $(3 - 2x)^3$. [4]

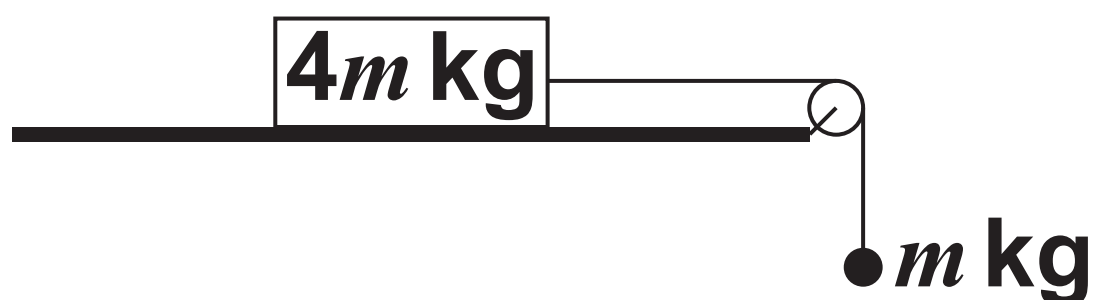
3 A particle is in equilibrium under the action of three forces in newtons given by

$$\mathbf{F}_1 = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \quad \mathbf{F}_2 = \begin{pmatrix} 2a \\ -3a \end{pmatrix} \quad \text{and} \quad \mathbf{F}_3 = \begin{pmatrix} 0 \\ b \end{pmatrix}.$$

Find the values of the constants a and b . [3]

- 4 Fig. 4 shows a block of mass $4m$ kg and a particle of mass m kg connected by a light inextensible string passing over a smooth pulley. The block is on a horizontal table, and the particle hangs freely. The part of the string between the pulley and the block is horizontal. The block slides towards the pulley and the particle descends. In this motion, the friction force between the table and the block is $\frac{1}{2}mg$ N.

Fig. 4



Find expressions for the acceleration of the system, the tension in the string. [4]

- 5** (i) Sketch the graphs of $y = 4 \cos x$ and $y = 2 \sin x$ for $0^\circ \leq x \leq 180^\circ$ on the same axes. [2]
- (ii) Find the exact coordinates of the point of intersection of these graphs, giving your answer in the form $(\arctan a, k\sqrt{b})$, where a and b are integers and k is rational. [4]
- (iii) A student argues that without the condition $0^\circ \leq x \leq 180^\circ$ all the points of intersection of the graphs would occur at intervals of 360° because both $\sin x$ and $\cos x$ are periodic functions with this period. Comment on the validity of the student's argument. [1]

6 In this question you must show detailed reasoning.

You are given that $f(x) = 4x^3 - 3x + 1$.

(i) Use the factor theorem to show that $(x + 1)$ is a factor of $f(x)$. [2]

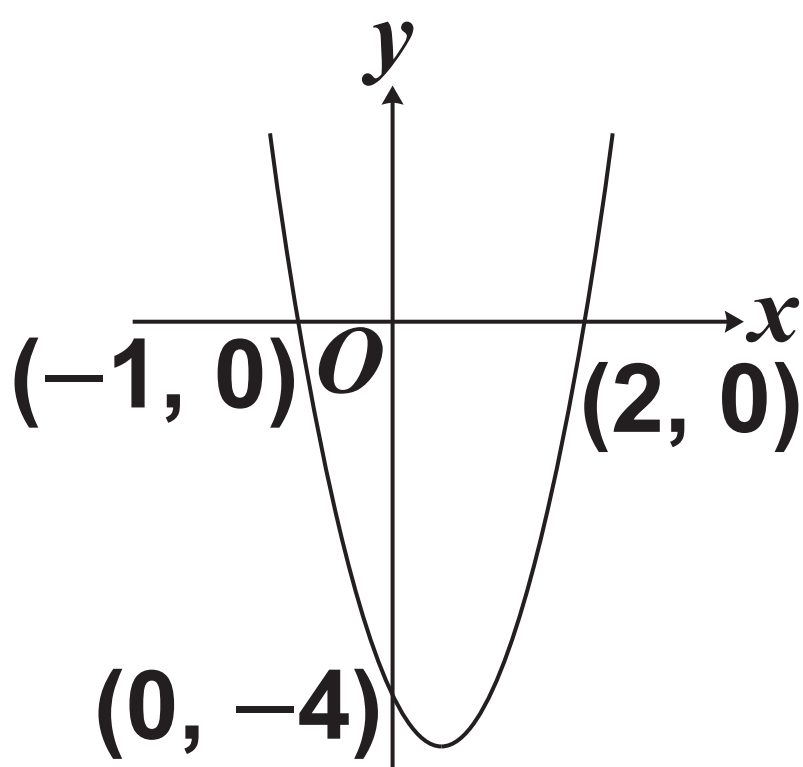
(ii) Solve the equation $f(x) = 0$. [3]

7 A toy boat of mass 1.5 kg is pushed across a pond, starting from rest, for 2.5 seconds. During this time, the boat has an acceleration of 2 m s^{-2} . Subsequently, when the only horizontal force acting on the boat is a constant resistance to motion, the boat travels 10 m before coming to rest. Calculate the magnitude of the resistance to motion. [6]

8 In this question you must show detailed reasoning.

Fig. 8 shows the graph of a quadratic function. The graph crosses the axes at the points $(-1, 0)$, $(0, -4)$ and $(2, 0)$.

Fig. 8



Find the area of the finite region bounded by the curve and the x -axis. [8]

9 The curve $y = (x - 1)^2$ maps onto the curve C_1 following a stretch scale factor $\frac{1}{2}$ in the x -direction.

(i) Show that the equation of C_1 can be written as $y = 4x^2 - 4x + 1$. [2]

The curve C_2 is a translation of $y = 4.25x - x^2$ by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$.

(ii) Show that the normal to the curve C_1 at the point $(0, 1)$ is a tangent to the curve C_2 . [7]

- 10 Rory runs a distance of 45 m in 12.5 s. He starts from rest and accelerates to a speed of 4 m s^{-1} . He runs the remaining distance at 4 m s^{-1} .**

Rory proposes a model in which the acceleration is constant until time T seconds.

- (i) Sketch the velocity-time graph for Rory's run using this model. [2]**
- (ii) Calculate T . [2]**
- (iii) Find an expression for Rory's displacement at time t s for $0 \leq t \leq T$. [2]**
- (iv) Use this model to find the time taken for Rory to run the first 4 m. [1]**

Rory proposes a refined model in which the velocity during the acceleration phase is a quadratic function of t . The graph of Rory's quadratic goes through $(0, 0)$ and has its maximum point at $(S, 4)$. In this model the acceleration phase lasts until time S seconds, after which the velocity is constant.

- (v) Sketch a velocity-time graph that represents Rory's run using this refined model. [1]**
- (vi) State with a reason whether S is greater than T or less than T . (You are not required to calculate the value of S .) [1]**

- 11 The intensity of the sun's radiation, y watts per square metre, and the average distance from the sun, x astronomical units, are shown in Fig. 11 for the planets Mercury and Jupiter.**

Fig. 11

	x	y
Mercury	0.3075	14 400
Jupiter	4.950	55.8

The intensity y is proportional to a power of the distance x .

- (i) Write down an equation for y in terms of x and two constants. [1]**
- (ii) Show that the equation can be written in the form $\ln y = a + b \ln x$. [2]**
- (iii) In the Printed Answer Booklet, complete the table for $\ln x$ and $\ln y$ correct to 4 significant figures. [2]**

- (iv) Use the values from part (iii) to find a and b . [3]**
- (v) Hence rewrite your equation from part (i) for y in terms of x , using suitable numerical values for the constants. [2]**
- (vi) Sketch a graph of the equation found in part (v). [2]**
- (vii) Earth is 1 astronomical unit from the sun. Find the intensity of the sun's radiation for Earth. [1]**

END OF QUESTION PAPER

BLANK PAGE

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.